

# THE CORE-PERIPHERY MODEL INCLUDING THE COMMERCIAL COSTS OF AGRICULTURAL GOOD

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## Abstract

*The purpose of this paper is to implement in real world the core-periphery model or the model if Paul Krugman's economic geography which proposed a congestion foundation of industrial activity in which agricultural activity was simplified to a maximum and a transport costs were ignored. Working methods used in this paper consist of the field theoretical approach in conjunction with mathematical modeling of the phenomenon – numerical simulation method, because the model is not solvable analytical, and graphical method, also. The results is precisely in the presentation of the impact of transport costs to agricultural goods on economic geography. By way of conclusion, we can see that using the transport costs does not alter the original model.*

**Key words:** the core-periphery model, agglomeration equilibrium, dispersion equilibrium

## INTRODUCTION

Because Paul Krugman's original model gives only a theoretical approach, operating with very strong simplification, and even with some ignorance, we considered it necessary to introduce a change to the initial model, so that it can be used in real condition of the economy<sup>1</sup>. The sequential approach can contribute to a generalization of the model in the sense of enlargement and the introduction of aspects that were initially ignored. The model will approach the real world in a much greater extent.

## MATERIAL AND METHODS OF WORK

### Initial model

The core-periphery model owing to Paul Krugman (1991) is based on a economy made of two regions, generically defined 1 and 2 and two areas, suppose to be the industry and the agriculture. Consumers from the specified areas are workers and farmers. The farmers earn a wage from the regional farms they are working for. They play a

double role, firstly as farmers and secondly as workers, that means that if they are owners of the farm, they become employees as well. The wage flow of the farmers is part of a bilateral transfer: they earn a wage from the farm's owner, so they work for the farm's owner. The farmers, the farm's owners, use the worker's work to produce agricultural goods in perfect competition with constant scale wage. Agricultural goods are sold to consumers from those two regions and these agricultural goods are supposed to have not transportation costs. In consideration of perfect competition's assumption, in this region is produced an agricultural good which is not differentiated. The labour used in this agriculture's sector is supposed to be fixed between regions.

Also, there is an industrial sector which has a certain number of companies, every of them produce a differentiated good, which means every of them produce a specified good and, for that, just labour factor is used on internal scale savings terms. These assumptions put companies in a monopolistic competition which affect price level for their products. The industrial goods are differentiated which means that every company from this region is producing a certain good, in fact a certain variety of industrial good. Transaction of industrial goods between regions suppose

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<sup>1</sup> Brakma S., Garretsen H. and van Marrewijk C., *An Introduction to Geographical Economics*, Cambridge University Press, 2003

transport costs. The labour from industrial sector is mobile between regions.

On these terms, we will have the following relations:

a) The general equation of income for “r” regions will be:

$$Y_r = \mu \lambda_r w_r + (1 - \mu) \Phi_r \quad (1)$$

in which  $\mu$  is a constant value which means the weight of consumers’ expense made by procuring industrial goods. Economics has a  $L_A$  farmers offer and every regions has a exogen weight from these agricultural labour  $\Phi_r$ . Industrial labour  $L_M$ , which is on a level with economics, is mobile between regions, so  $\lambda_r$  is the weight of “r” region in world-wide offer ( $L_M$ ). For relieving, there are used normalizations - choosing the following kind of units:  $L_M = \mu$ ;  $L_A = (1 - \mu)$ .  $w_r$  which means workers’ nominal wage from “r” region.

Particularly, for 2 regions, the equations will be:

$$Y_1 = \mu \lambda w_1 + \frac{(1-\mu)}{2} \quad (2);$$

$$Y_2 = \mu(1 - \lambda)w_2 + \frac{(1-\mu)}{2} \quad (3)$$

b) The general equation of industrial goods’ price index is:

$$G_r = [\sum_s \lambda_r (w_r T_{rs})^{1-\sigma}]^{\frac{\sigma}{\sigma-1}} \quad (4)$$

in witch sigma means substitution elasticity for any two industrial variety, coefficient which is supposed to be constant. In this realtion (4) transport cost is showed up too. The model suppose the transport cost to be “iceberg” type, which means that a unit of a good is sent from a region to another, only a part of it  $1/T$  will reach the destination, rest of it is “melting” on the way, whence this comparison with an iceberg. So, for these two regions the industrial goods’ price index will be:

$$G_1 = [\lambda w_1^{1-\sigma} + (1 - \lambda)(w_2 T)^{1-\sigma}]^{\frac{\sigma}{\sigma-1}} \quad (5)$$

$$G_2 = [\lambda(w_1 T)^{1-\sigma} + (1 - \lambda)w_2^{1-\sigma}]^{\frac{\sigma}{\sigma-1}} \quad (6)$$

c) The nominal wage equation is:

$$w_r = [\sum_s Y_s T_{rs}^{1-\sigma} G_s^{\sigma-1}]^{\frac{1}{\sigma}} \quad (7)$$

For a two regions economics we will have:

$$w_1 = [Y_1 G_1^{\sigma-1} + Y_2 G_2^{\sigma-1} T^{1-\sigma}]^{\frac{1}{\sigma}}; \quad (8)$$

$$w_2 = [Y_1 G_1^{\sigma-1} T^{1-\sigma} + Y_2 G_2^{\sigma-1}]^{\frac{1}{\sigma}} \quad (9)$$

d) The real wage equation will be:

$$\omega_r = w_r G_r^{-\mu} \quad (10)$$

$$\omega_1 = w_1 G_1^{-\mu}; \quad (11) \quad \omega_2 = w_2 G_2^{-\mu} \quad (12)$$

The deference between real wages from those two regions is bringing about the reason for industrial workers’ migration; in other words, those workers will leave for the region which offer the greatest real wage<sup>2</sup>.

The core-periphery model for two regions economics is build by 8 equations: (2), (3), (5), (6),(8), (9), (11) and (12); this model is required to be solved for two fixed dimensions of  $\mu$ , sigma parameters and for different levels of transport costs T. Solving these equations, we can obtain a relation between lambda, which means the weight of industrial labour from region 1 in total

<sup>2</sup>Fujita M., Krugman P. and Venables A.J., *The Spatial Economy*, The MIT Press, 2001;

industrial labour, and  $\omega_1/\omega_2$  which means the real relative wage.

Unfortunately, this system of equations can not be solved analytical, but only through numerical simulations, which means solving every level of lambda  $\in [0,1]$  in  $\omega_1/\omega_2$ .

It has been ascertained that for different dimensions of commercial costs, the relative real wage is changing significantly.

The model core-periphery including the commercial costs at agricultural good

Including the commercial costs at agricultural goods, in the circumstances in which a good is considered homogeneous and sold on a perfect competition market, it suppose an equality between the wages from this sector and prices, and, also, an unit of labour from this sector will produce an unit of good. As a result of including the

commercial costs, the wage and the prices won't be equivalent in those two regions because of the effect of the commercial costs, noticed with TA, and, for a facility understanding, the industrial costs will be noticed with TM. The differences between wages from this sector will belong to situation of the region. Situation means that region may be importer or exporter of that agricultural good.

Because of including the commercial costs at agricultural goods, the equations system of the core-periphery model will be:

$$\frac{wA_1}{wA_2} = TA \Rightarrow wA_1 = wA_2 TA \quad (13)$$

$$Y_1 = \mu \lambda wM_1 + \frac{1-\mu}{2} wA_1; \quad (14)$$

$$Y_2 = \mu(1-\lambda)wM_2 + \frac{1-\mu}{2} wA_2 \quad (15)$$

$$GM_1 = [\lambda(wM_1 TM)^{1-\sigma} + (1-\lambda)(wM_2 TM)^{1-\sigma}]^{\frac{1}{2-\sigma}}; \quad (16)$$

$$GM_2 = [\lambda(wM_1 TM)^{1-\sigma} + (1-\lambda)wM_2^{1-\sigma}]^{\frac{1}{2-\sigma}} \quad (17)$$

$$wM_1 = [Y_1 GM_1^{\sigma-1} + Y_2 GM_2^{\sigma-1} TM^{1-\sigma}]^{\frac{1}{\sigma}}; \quad (18)$$

$$wM_2 = [Y_1 GM_1^{\sigma-1} TM^{1-\sigma} + Y_2 GM_2^{\sigma-1}]^{\frac{1}{\sigma}} \quad (19)$$

$$\omega_1 = wM_1 GM_1^{-\mu} wA_1^{\mu-1}; \quad (20) \quad \omega_2 = wM_2 GM_2^{-\mu} wA_2^{\mu-1} \quad (21)$$

It can be seen that when TA=1, the equation system (13)-(21) is reduced according to initial core-periphery model.

Numerical assumption

We solve the system core-periphery, considering sigma =5 and  $\mu=0,4$  and for different dimensions of transport costs for industrial and agricultural goods. The

numerical assumptions were done with Maple 12 software. We can place the software at your disposal if you are interested in it. So, if TA=1 and  $wA_1=wA_2=1$ , will generate the initial core-periphery model.

The most important figures are:

Figure no.1. Model core-periphery  
 $\sigma=5$   $\mu=0.4$   
 transp costs industrial goods  $T=1,5$   
 transp costs agricultural goods 1,05

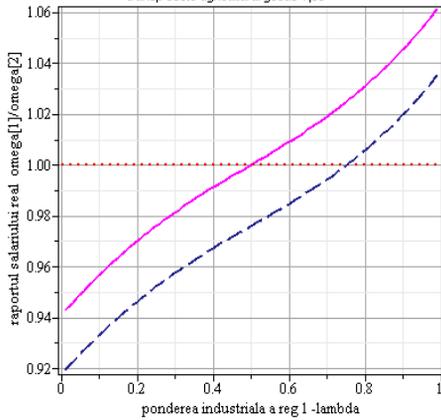


Figure no.2. Model core-periphery  
 $\sigma=5$   $\mu=0.4$   
 transp costs industrial goods  $T=1,5$   
 transp costs agricultural goods 1,1

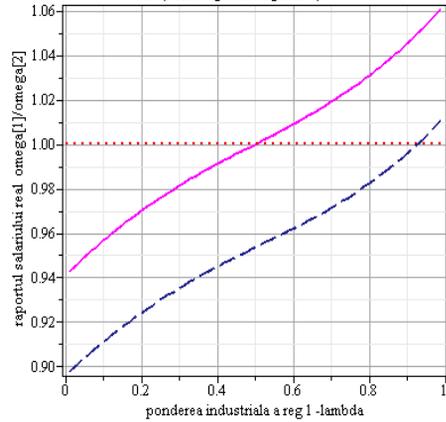


Figure no.3. Model core-periphery  
 $\sigma=5$   $\mu=0.4$   
 transp costs industrial goods  $T=1,5$   
 transp costs agricultural goods 0,95

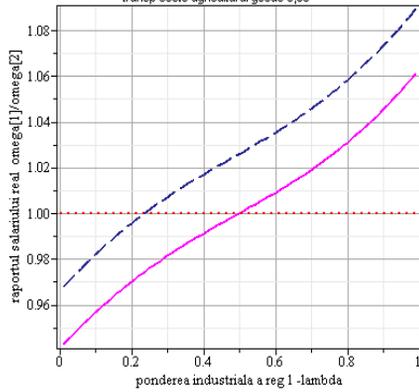


Figure no.4. Model core-periphery  
 $\sigma=5$   $\mu=0.4$   
 transp costs industrial goods  $T=1,7$   
 transp costs agricultural goods 1,05

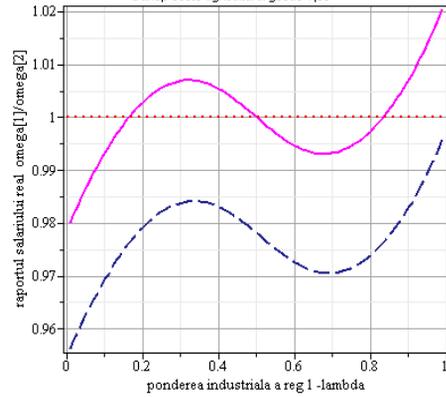


Figure no.5. Model core-periphery  
 $\sigma=5$   $\mu=0.4$   
 transp costs industrial goods  $T=1,7$   
 transp costs agricultural goods 1,1

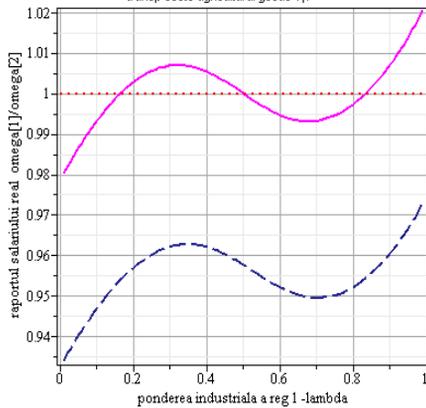
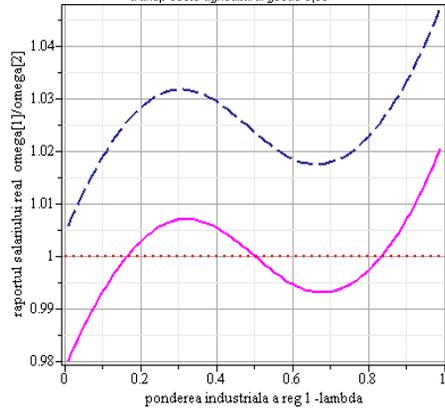
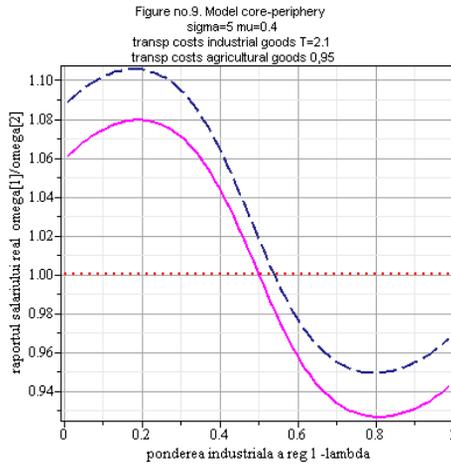
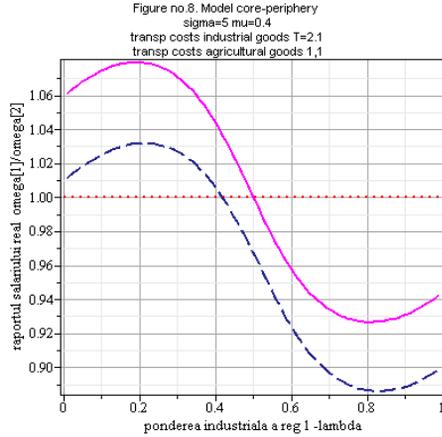
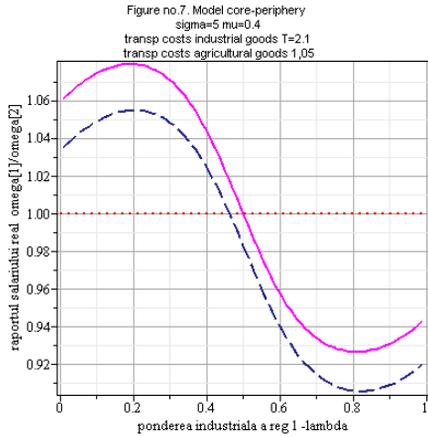


Figure no.6. Model core-periphery  
 $\sigma=5$   $\mu=0.4$   
 transp costs industrial goods  $T=1,7$   
 transp costs agricultural goods 0,95





## RESULTS AND DISCUSSION

As you can see, have been used three levels of transport costs for industrial goods  $TM = 1,5$ ,  $TM = 1,7$  and  $TM = 2,1$ . We mention that in all figures with continuous line we designated core-periphery model ignoring transport costs of agricultural goods, in other words, Paul Krugman's original model, while we designated by dashed model including these costs, their level being shown in each graph.

So, 1-3 Figures have been compiled under a transport cost to small industrial product  $TM = 1,5$ , generating the relative real wage to have a slope ascending to the share of workers in region 1, designated by  $\lambda$ . If the labour is equally divided

between the two regions,  $\lambda$  is equal to 0,5, the relative real wage is equal to 1, there is an equilibrium dispersion. Any enlargement of  $\lambda$  will contribute to increasing the real wage in region 1. This increase continues until the  $\lambda$  is equal to 1, and, also, until the whole activity will be concentrated in region 1. It will become the "core" while region 2 will cease to exist, this region becoming the "periphery". Be noted that the introduction of transport costs maintains agricultural goods model behavior, but alter the balance of dispersion when the relative real wage is equal to 1 depending

on the size of transport costs on agricultural goods<sup>3</sup>.

Increasing share of industrial workers will rise to an increase of real relative wage and this process continues until the entire commercial activity will be concentrated in one region which becomes the core, and other periphery. Then, we analyze the case when industrial goods' trade costs are large,  $TM = 2,1$ , as in graphs 7-9. Be noted that relative real wage register a slope descending to the share of workers in region 1, depending on  $\lambda$ . This situation to determine the relative real wage is supraunitar if  $\lambda$  is less than 0.5 and subunitar if  $\lambda$  is greater than 0.5. This means that if one region share a register of industrial labor greater than 0.5 it becomes less attractive to workers than other regions. In such a situation the economy is heading towards a symmetric equilibrium in which labor force is equally divided between regions. Introduction of transport costs for agricultural goods maintain the behavior of the model, but change the balance directly depending on the size of these costs on agricultural goods. For an intermediate level of transport costs  $TM = 1,7$  on industrial goods, respectively in figures 4-6, to register a stable symmetric equilibrium in the sense that if  $\lambda$  is equal to 0.5 and subsequently boost  $\lambda$  would result in a relative real wage reduction and they will return to the page. But beyond this equilibrium there are two unstable equilibria in which real wage curve intersects the real income relative right which is equal to 1. If  $\lambda$  starts from a value small enough or large enough, the economy will converge to the symmetric equilibrium, but by a concentration of industry in a certain region, which will create a core in one of the region and a periphery in the other. Introduction of transport costs at agricultural goods eliminate symmetrical balance and industrial focuses only in one region.

## OPINION

By way of conclusion it may be mentioned that the introduction of transport costs do not change the fundamental behavior of core-periphery model, but this approach was achieved by an extension of the original model's Paul Krugman. Literature in this field aims and other uses of extensions, such as a greater number of regions, including monopolistic competition and differentiated agricultural goods, of asymmetric transport costs, etc. For this reason in the future we propose a general approach of the model in which to study various private aspects.

## REFERENCES

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<sup>3</sup>Krugman, P.: *Increasing returns and economic geography*, Journal of Political Economy, 1991, 99, pag. 483-499